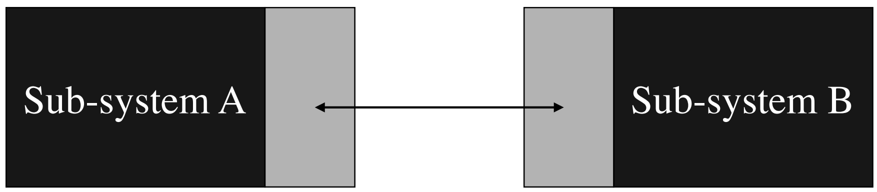
Formal Specification

Approaches to Formal Specification

- Functional
  - The system is described as a number of functions.
  - Unnatural and complex for large systems
- Algebraic
  - The system is described in terms of operations and their relationships.
- Model-based
  - A model of the system is constructed using well-understood mathematical entities such as sets and sequences

Algebraic Specification

- Based on concept of abstract data types
- Large systems are usually decomposed into sub-systems which are accessed through a defined *interface*.



Algebraic Specification (2)

Components of a specification:

- Specification name and generic parameter list
- Type name and declaration of imported specifications
- Informal description
- Operation signatures
- Axioms that define the operational semantics

Array Specification

ARRAY (Elem: [Undefined → Elem])

sort Array
imports Integer

Arrays are collections of elements of generic type Elem. They have a lower and upper bound…

```
Create(Integer, Integer) → Array
Assign(Array, Integer, Elem) → Array
First(Array) → Integer
Last(Array) → Integer
Eval(Array, Integer) → Elem
```

Array Specification Axioms

```
First(Create(x, y)) = x
First(Assign(a, n, v)) = First(a)
Last(Create(x, y)) = y
Last(Assign(a, n, v)) = Last(a)
Eval(Create(x, y), n) = Undefined
Eval(Assign(a, n, v), m) =
      if m < First(a) or m > Last(a) then Undefined
      else if m = n then v
      else Eval(a,m)
```

```
Eval(
  Assign(
    Assign(
      Create(1,10), 1, 5), 2, 10), 1) = 5
```

Binary Tree Specification

Signatures:

```
Create → BT
Add(BT, Elem) → BT
Left(BT) → BT
Right(BT) → BT
Data(BT) → Elem
Empty(BT) → Boolean
Contains(BT, Elem) → Boolean
Build(BT, Elem, BT) → BT
```

Binary Tree Specification (2)

Axioms:

```
Add(Create, E) = Build(Create, E, Create)
Add(B, E) = if E < Data then Add(Left(B), E)
            else Add(Right(B), E)
Left(Create) = Create
Data(Create) = Undefined
Left(Build(L, D, R)) = L
Data(Build(L, D, R)) = D
Empty(Create) = true
Empty(Build(L, D, R)) = false
Contains(Create, E) = false
Contains(Build(L, D, R), E) =
   if E = D then true else
     if E < D then Contains(L, D) else
  Contains(R,D)
```

Systematic Specification

- Specification structuring
  - The informal interface specification is structured into a set of abstract data types with proposed operations.
- Specification naming
  - Name, generic parameters, type
- Operation selection
  - Constructors, Manipulators, Inspectors
- Informal operation specification
- Syntax definition
- Axiom definition

Structured Specification: Reuse

- Instantiation of generic specifications
  - Array<Integer>, Array<Array<Character>>, etc.
- Incremental development of specifications
  - Import specification of simple types and use them in the definition of more complex types, e.g. the specification of Coordinate might be used in the specification of Cursor.
- Enrichment of specifications
  - Inheritance of signatures and axioms from a base specification

Error Specification

Several approaches:

- A special constant operation, such as *Undefined*, may be defined.
- Operations may be defined to evaluate to *tuples,* where one element of the tuple is a boolean that indicates successful evaluation.
- The specification may include an exceptions section which defines conditions under which the axioms do not hold.

Model-based Specification with Z

Formal specification of software by developing a mathematical model of the system

Objectives

- To introduce an approach to formal specification based on mathematical system models
- To present some features of the Z specification language
- The illustrate the use of Z using small examples
- To show how Z schemas may be used to develop incremental specifications

Model-based Specification

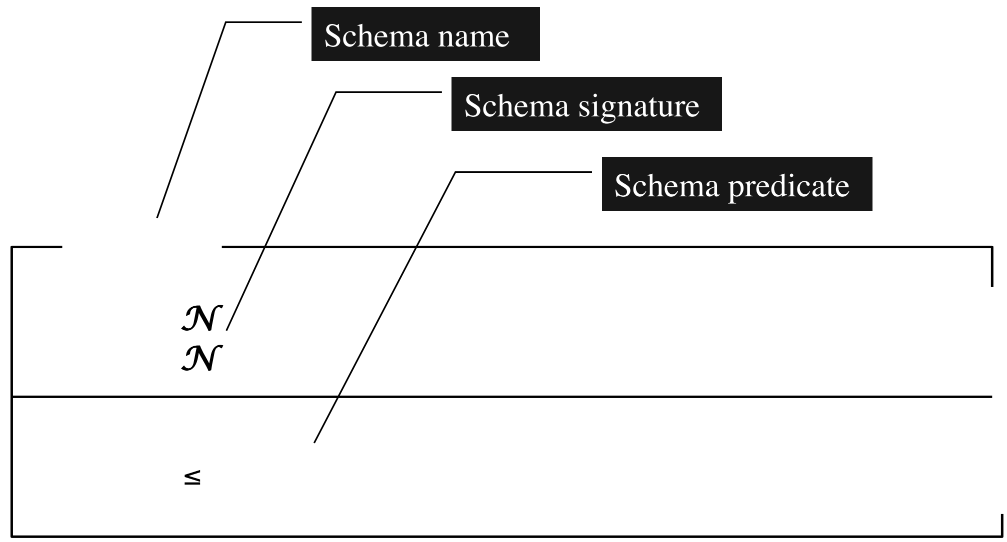
- Defines a model of a system using well-understood mathematical entities such as sets and functions
- The state of the system is not hidden (unlike algebraic specification)
- State changes are straightforward to define

Z Specification Language

- Based on typed set theory
- Probably now the most widely-used specification language
- Includes schemas, an effective low-level structuring facility
- Schemas are specification building blocks
- Graphical presentation of schemas make Z specifications easier to understand

Z Schemas

- Introduce specification entities and defines invariant predicates over these entities
- A schema includes
  - A name identifying the schema
  - A signature introducing entities and their types
  - A predicate part defining invariants over these entities
- Schemas can be included in other schemas and may act as type definitions
- Names are local to schemas



An *Indicator* Specification

Indicator

light: { off, on }
reading: $\mathcal{N}$
danger_level: $\mathcal{N}$

---

light = on ⇔ reading ≤ danger_level

Storage Tank Specification

Storage_tank

Container
Indicator

---

reading = contents
capacity = 5000
danger_level = 50

Complete Storage Tank Specification

Storage_tank

contents: $\mathcal{N}$
capacity: $\mathcal{N}$
light: { off, on }
reading: $\mathcal{N}$
danger_level: $\mathcal{N}$

---

contents $\leq$ capacity
light = on $\Leftrightarrow$ reading $\leq$ danger_level
reading = contents
capacity = 5000
danger_level = 50

Z Conventions

- A variable name decorated with a dash (quote mark), N′, represents the value of the state variable N after an operation
- A schema name decorated with a dash introduces the dashed values of all names defined in the schema
- A variable name decorated with a ! represents an output

Z conventions

- A variable name decorated with a ? represents an input
- A schema name prefixed by the Greek letter Xi (Ξ) means that the defined operation does not change the values of state variables
- A schema name prefixed by the Greek letter Delta (Δ) means that the operation changes some or all of the state variables introduced in that schema

Operation Specification

- Operations may be specified incrementally as separate schema then the schema combined to produce the complete specification
- Define the ‘normal’ operation as a schema
- Define schemas for exceptional situations
- Combine all schemas using the disjunction (or) operator

A partial spec. of a fill operation

Fill_OK

ΔStorage_tank
amount?: $\mathcal{N}$

---

contents + amount? $\leq$ capacity
contents’ = contents + amount?

Storage tank fill operation

OverFill

ΞStorage_tank
amount?: $\mathcal{N}$
r!: seq CHAR

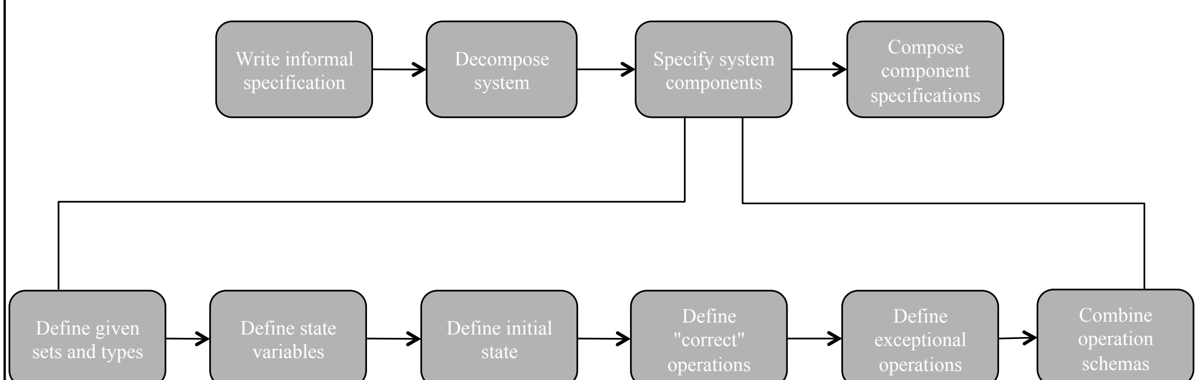
---

capacity < contents + amount?
r! = “Insufficient tank capacity – Fill canceled”

Fill

Fill_OK ∨ OverFill

The Z Specification Process



Data dictionary specification

- A data dictionary will be used as an example. This is part of a CASE system and is used to keep track of system names
- Data dictionary structure
  - Item name
  - Description
  - Type. Assume in these examples that the allowed types are those used in UML Class Diagrams
  - Creation date

Data Dictionary Modeling

- A data dictionary may be thought of as a mapping from a name (the key) to a value (the description in the dictionary)
- Operations are
  - Add. Makes a new entry in the dictionary or replaces an existing entry
  - Lookup. Given a name, returns the description.
  - Delete. Deletes an entry from the dictionary
  - Replace. Replaces the information associated with an entry

Given Sets

- Z does not require everything to be defined at specification time
- Some entities may be 'given' and defined later
- The first stage in the specification process is to introduce these given sets
  - [NAME, DATE]
  - We don't care about these representations at this stage

Type Definitions

- There are a number of built-in types (such as INTEGER) in Z
- Other types may be defined by enumeration
  - ModelElement = { class, field, method, association }
- Schemas may also be used for type definition. The predicates serve as constraints on the type

Data Dictionary Entry

DataDictionaryEntry

name: NAME
description: seq char
type: ModelElement
creation_date: DATE

#description ≤ 2000

Specification Using Functions

- A function is a mapping from an input value to an output value, e.g:

  SmallSquare = {1→1, 2→4, 3→9, 4→16, 5→25, 6→36, 7→49 }

  SmallSquare = { (1,1), (2,4), (3,9), (4,16), (5,25), (6,36), (7,49) }

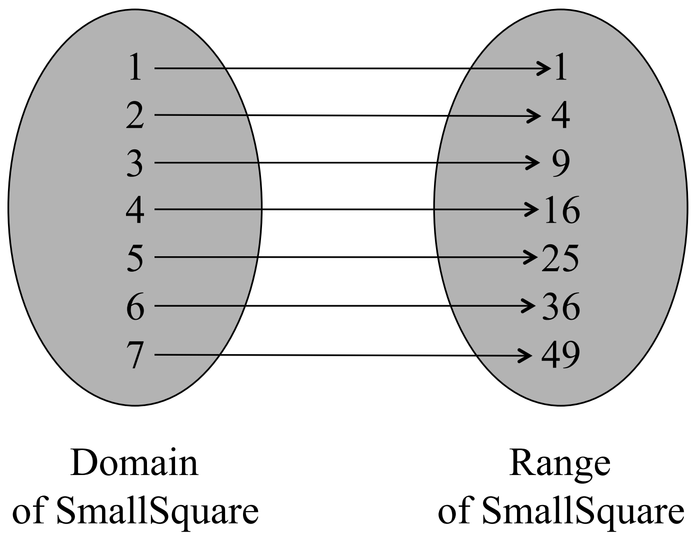
- The domain of a function is the set of inputs over which the function has a defined result

  Domain SmallSquare = {1, 2, 3, 4, 5, 6, 7 }

- The range of a function is the set of results which the function can produce

  Range SmallSquare = {1, 4, 9, 16, 25, 36, 49 }

SmallSquare Function



Set and Function Notation

- We can use set and function notations interchangeably, depending on what is more convenient.
- Function notation:
  SmallSquare(3) = 9
- Set notation:
  (3, 9) ∈ SmallSquare

Data dictionary as a function

DataDictionary

DataDictionaryEntry
ddict: NAME $\mapsto$ { DataDictionaryEntry }

Data dictionary initialization

Init_DataDictionary

DataDictionary′

---

ddict′ = { }

Add and lookup operations

Add_OK

Δ DataDictionary
entry?: DataDictionaryEntry

---

entry?.name $\notin$ dom ddict
ddict′ = ddict $\cup$ { entry?.name $\rightarrow$ entry? }

Lookup_OK

Ξ DataDictionary
name?: NAME
entry!: DataDictionaryEntry

---

name? $\in$ dom ddict
entry! = ddict(name?)

Add and lookup operations

Add_Error

Ξ DataDictionary
entry?: DataDictionaryEntry
error!: seq char

---

entry?.name $\in$ dom ddict
error! = “Name already in dictionary”

Lookup_Error

Ξ DataDictionary
name?: NAME
error!: seq char

---

name? $\notin$ dom ddict
error! = “Name not in dictionary”

Function over-riding operator

- ReplaceEntry uses the function overriding operator (written ⊕). This adds a new entry or replaces and existing entry.
  - phone = { Ian → 3390, Ray → 3392, Steve → 3427}
  - The domain of phone is {Ian, Ray, Steve} and the range is {3390, 3392, 3427}.
  - newphone = {Steve → 3386, Ron → 3427}
  - phone ⊕ newphone = { Ian → 3390, Ray → 3392, Steve → 3386, Ron → 3427}

Replace operation

Replace_OK

Δ DataDictionary
entry?: DataDictionaryEntry

---

entry?.name ∈ dom ddict
ddict′ = ddict ⊕ { entry?.name → entry? }

Deleting an Entry

- Uses the domain subtraction operator (written $\lhd$) which, given a name, removes that name from the domain of the function

$$phone = \{ Ian \rightarrow 3390, Ray \rightarrow 3392, Steve \rightarrow 3427\}$$
$$\{Ian\} \lhd phone = \{Ray \rightarrow 3392, Steve \rightarrow 3427\}$$

Delete Entry Operation

Delete_OK

Δ DataDictionary
name?: NAME

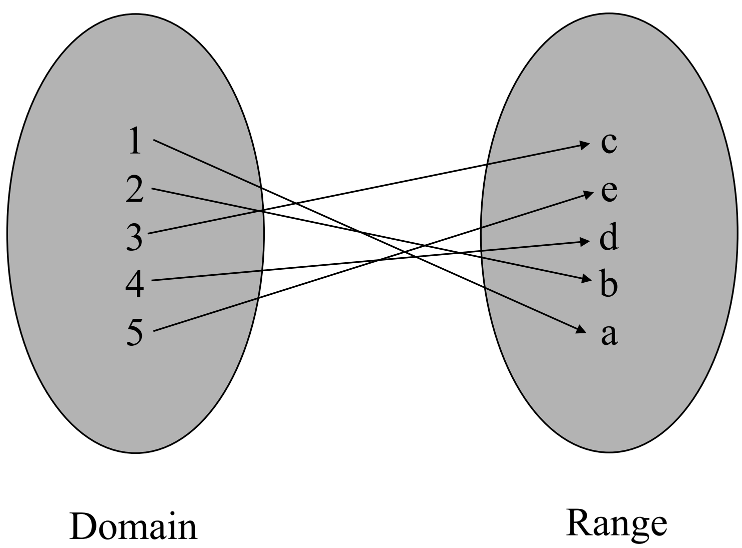
---

name? $\in$ dom ddict
ddict′ = { name? } $\lhd$ ddict

Specifying Ordered Collections

- Specification using sets does not allow ordering to be specified
- Sequences are used for specifying ordered collections
- A sequence is a function mapping consecutive integers to associated values

A Z Sequence



{ (1,a), (2,b), (3,c), (4,d), (5,e) }

Extract Operation

- The Extract operation extracts from the data dictionary all those entries whose type is the same as the type input to the operation
- The extracted list is presented in alphabetical order
- A sequence is used to specify the ordered output of Extract

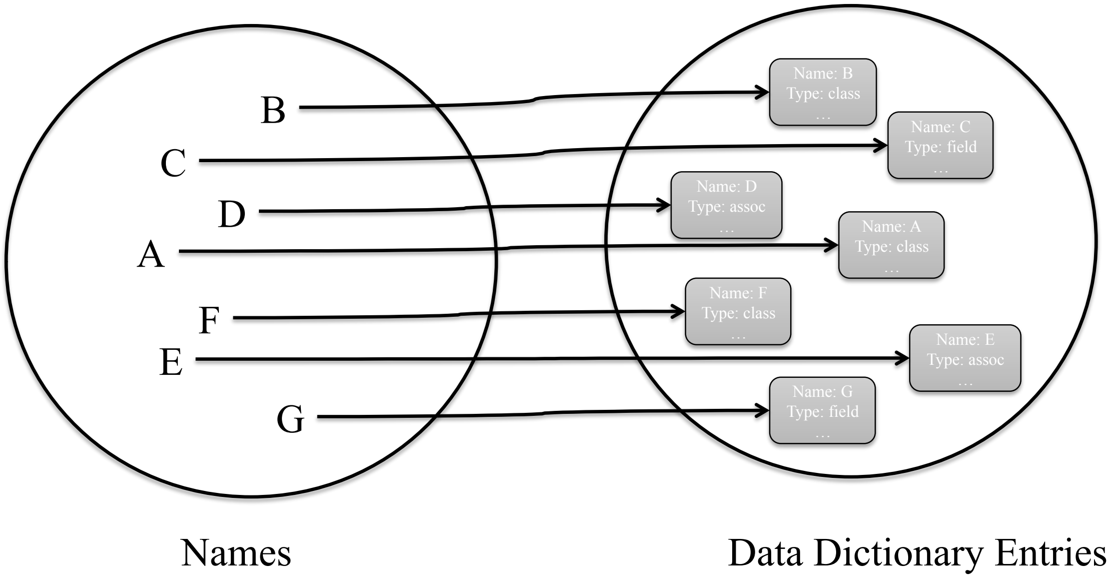
The Extract operation

Extract

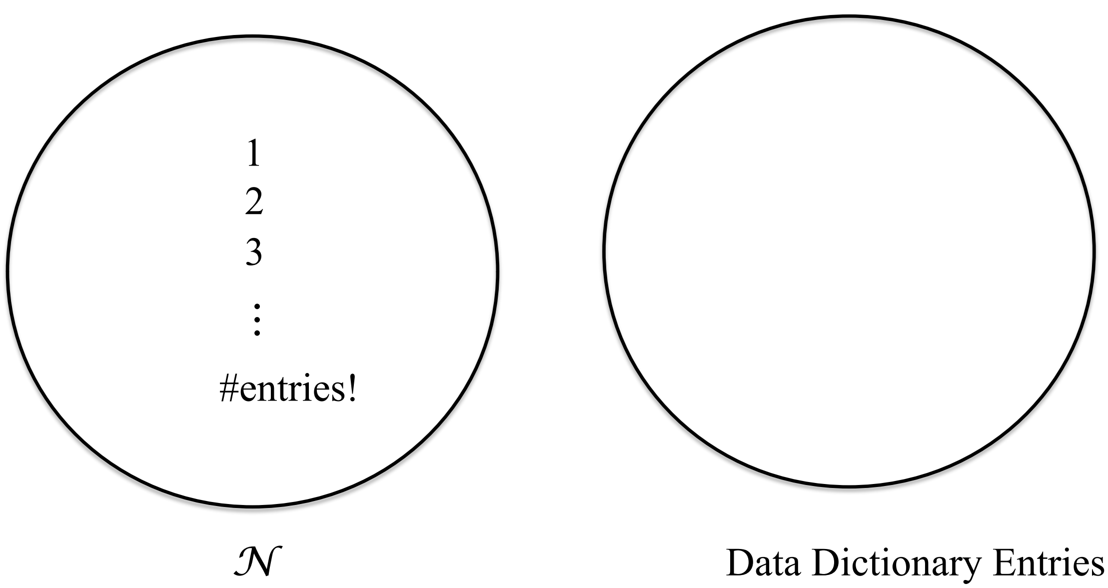
DataDictionary
entries!: seq DataDictionaryEntry
type?: ModelElement

$\forall n \in \text{dom ddict: ddict}(n)\text{.type} = \text{type?} \Rightarrow \text{ddict}(n) \in \text{rng entries!}$
$\forall 1 \leq i \leq \#\text{entries!: entries!}(i)\text{.type} = \text{type?}$
$\forall 1 \leq i \leq \#\text{entries!: entries!}(i) \in \text{rng ddict}$
$\forall i,j \in \text{dom entries!: } (i < j) \Rightarrow \text{entries!}(i)\text{.name} <_{\text{NAME}} \text{entries!}(j)\text{.name}$

ddict

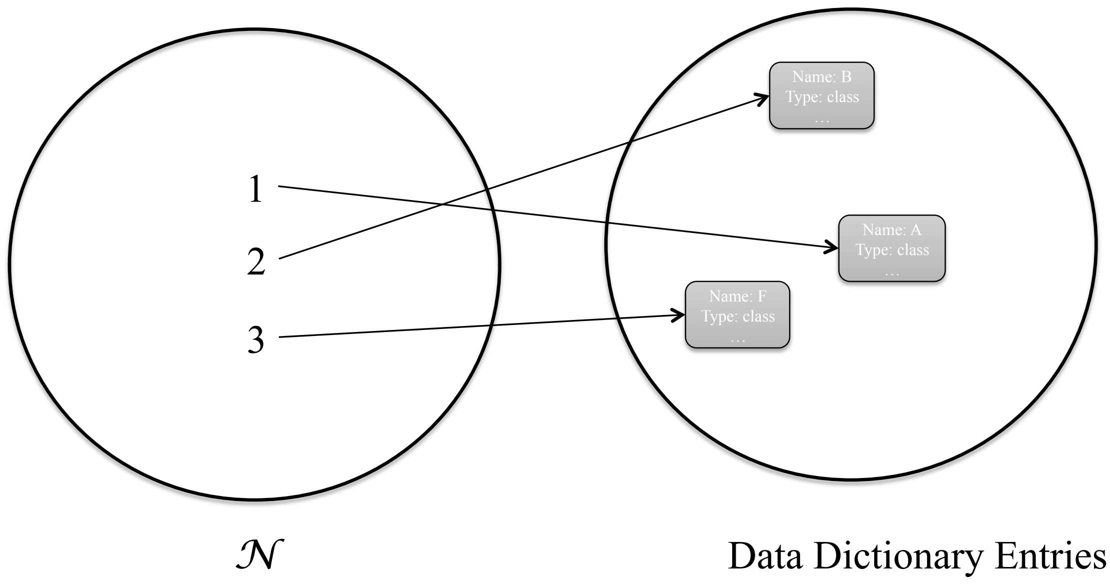


entries!



We need invariants to define the range and the mapping.

entries!



Example: type? = class

Extract predicate

- For all entries in the data dictionary whose type is *type?*, there is an entry in the output sequence
- The type of all members of the output sequence is *type?*
- All members of the output sequence are members of the range of ddict
- The output sequence is ordered by entry name

Data dictionary specification

The_Data_Dictionary

DataDictionary
Init_DataDictionary
Add
Lookup
Delete
Replace
Extract

Key points

- Model-based specification relies on building a system model using well-understood mathematical entities
- Z specifications are made up of mathematical model of the system state and a definition of operations on that state
- A Z specification is presented as a number of schemas
- Schemas may be combined to make new schemas

Key points

- Operations are specified by defining their effect on the system state. Operations may be specified incrementally then different schemas combined to complete the specification
- Z functions are a set of pairs where the domain of the function is the set of valid inputs. The range is the set of associated outputs. A sequence is a special type of function whose domain is the consecutive integers

Objectives

- What is the role of formal software specification in the software process?
- What are the pros and cons of formal specification?
- When can formal specification be cost effective?

Software Process

- As specification is developed in detail, understanding of the specification increases. Creating a formal specification usually reveals errors in the informal specification, that are fed back to allow correction.

Why no formal specifications?

- Management is unwilling to adopt techniques without an obvious payoff.
- Software engineers lack training in discrete math and logic.
- Customers are unfamiliar with formal techniques.
- Certain classes of software systems are difficult to specify using existing techniques.
- Practicality of techniques is not well known.
- Little effort has been devoted to method and tool support for formal techniques.

Why no formal specifications? (2)

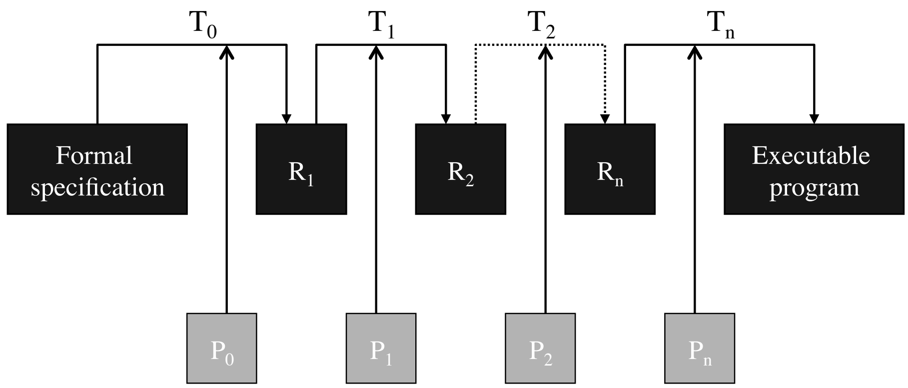
- The move to interactive systems
  - Most formal specification languages are unable to cope with interactive graphical interfaces.
  - Difficult to integrate formal methods with rapid prototyping.
- Successful software engineering
  - If major progress is being made in improving software quality without using formal methods, why should they be introduced now?

Pros of formal specification

- Provides insight into understanding requirements and the software design.
  - Reduces requirement errors.
- May be analyzed using mathematical methods.
  - Ability to reason about the specification
- May be automatically processed.
  - Development and debugging tools
- May guide testing.
  - Identifying appropriate test cases

Transformational development

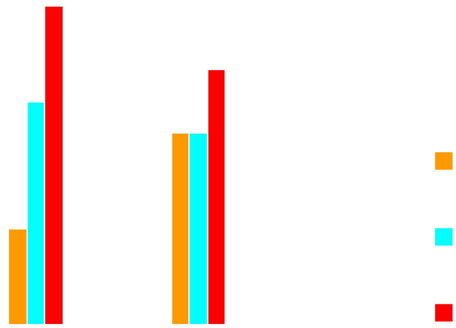
- Easier to prove correctness *incrementally*:



Myths of formal methods (Hall 1990)

- Formal methods result in perfect software
- Formal methods mean program proving
- Formal methods increase development costs
- Formal methods require a high level of mathematical skill
- Customers cannot understand formal specifications
- Formal methods have only been used for trivial system development

Development costs



Verdict on formal specification

- For a large class of systems, particularly highly interactive systems, formal specification is probably not cost effective for the foreseeable future.
- Where system dependability (safety, reliability, security, etc.) is critical, formal methods will probably become common.
- Formal specifications may also be very useful in defining *standards* where precision and unambiguity are essential.